Opportunistic Medium Access Control and Routing on a Poisson Wireless Network

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Structure of the Talk

- **Wireless channels and SINR**
- **Medium Access Control**
  - **Spatial Aloha**
  - **Opportunistic Aloha**
- **Routing**
  - **Near neighbor routing**
  - **Opportunistic routing**

CAPACITY OF THE ADDITIVE WHITE GAUSSIAN NOISE CHANNEL

- **On a shared wireless medium, the capacity of a channel between a transmitter and a receiver is given by Shannon's theorem**

$$\theta = B \log(1 + \mathbf{SINR}) \quad \textbf{where} \quad \mathbf{SINR} = \frac{R}{W + I}$$

– R **power with which the signal is received;**

– W **power of thermal noise at the receiver;**

– I **power of interference at the receiver.**

- **Hence** $\theta \geq K$ **iff** $\mathbf{SINR}=\frac{R}{W+I} \geq T = f(K)$**.**

INTERFERENCE IN A POISSON FIELD OF NODES

– $\Phi = \{X_i\}$ **homogeneous Poisson point process on** $\mathbb{R}^2$ **representing location of transmitters;**

– $l : \mathbb{R}^+ \to \mathbb{R}^+$: **attenuation function:**
for instance $l(r) = r^\beta$ **with** $\beta > 2$ **(power law model);**

– $F_{i,x} \in \mathbb{R}^+$: **fading from** X_i **to** $x \in \mathbb{R}^2$**, i.i.d. and ind. of** Φ**.**

- **Interference field: Poisson shot noise field**

$$I_\Phi(x) = \sum_i \frac{F_{i,x}}{l(|x - X_i|)}$$

a.s. finite provided $\mathbb{E}(F) < \infty$ **and** $\int l^{-1}(r) r dr < \infty$**, with LT**

$$\mathcal{L}_I(s) = \exp\left\{ -2\pi\lambda \int_0^\infty r \Big(1 - \mathcal{L}_F\big(s/l(r)\big)\Big)\, dr \right\}.$$

SPATIAL ALOHA PRINCIPLES

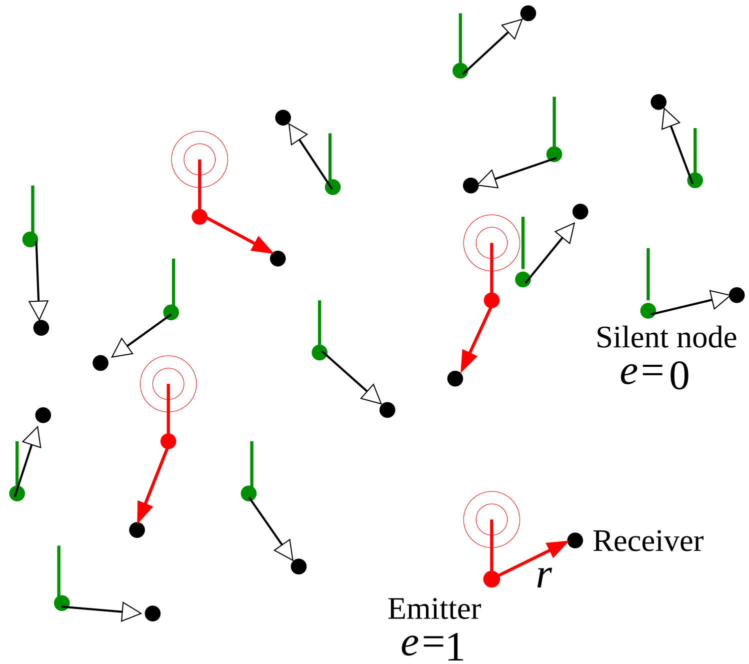
- **Data: wireless network made of nodes (potential transmitters) which form some realization of a Poisson point process in the Euclidean plane.**
- **Slotted version of Spatial Aloha:**
  - **Each node tosses a coin with bias p (MAP) to be allowed to access the medium**
  - **This creates a random exclusion area around each location**
  - **Success of transmission is decided in function of the SINR at the receiver of each effective transmitter.**

SINR MODEL FOR SPATIAL ALOHA

- $\Phi = \{X_i, (F_{i,.})\}$ **independently marked PPP**
- $e_i \in \{0, 1\}$**: right for** i **to access medium in current slot**
- $I_{\Phi^e}(y) = \sum_{i \neq 0} F_{i,y} e_i / l(|y - X_i|)$**: 'filtered' interference at** y**,**
- **Node at** 0 **active in slot can be received by that located at** y **iff**

$$\frac{F_{0,y}/l(|y|)}{W + I_{\phi^e}(y)} \geq T.$$

BIPOLAR ALOHA NETWORK



Each node has its receiver at some fixed distance r in some uniformly and independently sampled direction.

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/M CASE

- **Assumptions**
  - Φ **Poisson with intensity** λ
  - e_i **i.i.d. with** $P(e_i = 1) = p \Rightarrow \Phi^e$ **Poisson with intensity** λp**.**
  - **Rayleigh fading:** $F_{i,j}$ **i.i.d. exponential with mean** μ^{-1}
- **The probability of a successful transmission/coverage at distance** r **is**

$$
\begin{aligned}
p_r(\lambda p) &= \mathbb{P}(F \geq Tl(r)(W + I_{\Phi^e})) = \mathcal{L}_W\left(\mu Tl(r)\right)\mathcal{L}_{I^e}\left(\mu Tl(r)\right) \\
&= \mathcal{L}_W\left(\mu Tl(r)\right) \exp\left\{-2\pi\lambda p \int_0^\infty \frac{u}{1 + l(u)/(Tl(r))} du\right\}.
\end{aligned}
$$

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/M CASE (*continued*)

- **In the power law attenuation case with $W \equiv 0$,**

$$p_r(\lambda p) = \exp(-\lambda p r^2 T^{2/\beta} K(\beta)) \,,$$

where

$$K(\beta) = \frac{2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)}{\beta} = \frac{2\pi^2}{\beta \sin(2\pi/\beta)} \,.$$

BEST MAP GIVEN RANGE

- **Question: what value of p maximizes the spatial density of successful transmissions in the bipolar model with dist. r.**
- **Answer from maximization of $\lambda p_r(\lambda)$ spatial density of successful access to channel w.r.t. λ.**

$$\lambda_{\max} = \frac{1}{2\pi \int_0^\infty \frac{u}{1+l(r)/(Tl(u))} du}$$

and

$$\lambda_{\max} p_r(\lambda_{\max}) = e^{-1} \lambda_{\max} \mathcal{L}_W \left(\frac{\mu T}{l(r)} \right)$$

BEST MAP GIVEN RANGE (*continued*)

- **In the power law attenuation case,**
  - **Optimal spatial intensity of channel access**

$$\lambda_{\max} = \frac{1}{r^2 T^{2/\beta} K(\beta)}$$

  - **Optimal spatial intensity of successful transmissions**

$$\lambda_{\max} p_r(\lambda_{\max}) = \frac{1}{r^2 T^{2/\beta} e K(\beta)}$$

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/G CASE

- **Assumptions**
  - Φ **Poisson with intensity** λ**;**
  - $F_{i,j}$ **i.i.d. with general distribution with mean** μ^{-1}**;**
  - **Power law attenuation model;** $W \equiv 0$**.**
- $p_r(\lambda)$: **probability of a successful transmission at distance** r
- $\bar{p}_r$: **value of** $p_r(1)$ **for the model with power law attenuation function,** $T \equiv 1$ **and normalized fading** $\bar{F}_{i,j} = \mu F_{i,j}$**.**

LEMMA $p_r(\lambda) = \bar{p}_{rT^{1/\beta}\lambda^{1/2}}$.

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/G CASE (*continued*)

- **Optimization: for the power law attenuation function and** $W \equiv 0$

$$\lambda_{\max} = \frac{c_1}{r^2 T^{2/\beta}},$$

$$\lambda_{\max} p_r(\lambda_{\max}) = \frac{c_2}{r^2 T^{2/\beta}},$$

where the constants c_1, c_2 **do not depend on** r, T, μ**, provided** $\lambda_{\max}$ **is well defined.**

- **Proof: from**

$$\lambda p_r(\lambda) = \lambda \bar{p}_{rT^{1/\beta}\lambda^{1/2}} = \frac{1}{r^2 T^{2/\beta}} \lambda r^2 T^{2/\beta} \bar{p}_{rT^{1/\beta}\lambda^{1/2}}$$

$$c_1 = \mathbf{maxarg}_{\lambda \geq 0}\{\lambda \bar{p}_{\lambda^{1/2}}\}, \qquad c_2 = \max_{\lambda \geq 0}\{\lambda \bar{p}_{\lambda^{1/2}}\}.$$

PROBABILITY OF TRANSMISSION SUCCESS FOR GIVEN RANGE: M/G CASE (*continued*)

- **Fourier representation of the probability of coverage**

LEMMA If

– F **has a finite first moment and a square integrable density;**

– **Either** I^e **or** W **admit a density which is square integrable, then**

the probability of a successful transmission at distance r **is**

$$p_r(\lambda p) = \int_{s=-\infty}^{\infty} \mathcal{L}_{I^e}\left(2i\pi l(r)Ts\right) \mathcal{L}_W\left(2i\pi l(r)Ts\right) \frac{\mathcal{L}_F(-2i\pi s) - 1}{2i\pi s} ds\,.$$

with

$$\mathcal{L}_{I^e}(s) = \exp\left\{-2\pi\lambda p \int_0^{\infty} r\Big(1 - \mathcal{L}_F\big(s/l(r)\big)\Big)\, dr\right\},$$

OPPORTUNISTIC ALOHA

- **Spatial Aloha: selects the nodes allowed to transmit at random**
- **Opportunistic Aloha: selects the nodes allowed to transmit in function of their channel condition: the medium access indicator e_i of node i is**

$$e_i = \mathbf{1}(F_{i,i} > \theta),$$

where where θ is some deterministic threshold.

OPPORTUNISTIC ALOHA (*continued*)

LEMMA **Consider a bipolar Poisson network using Opportunistic Aloha with MAC threshold θ. Assume that**

- **The random variable $\widehat{F}$ (with law that of F conditional on $F > \theta$) has a finite first moment and a density which is square-integrable;**
- **Either W or I^e has a density which is square-integrable.**

Then

$$\widehat{p}_r(\lambda p) = \int_{\mathbb{R}} \mathcal{L}_{I^e}(2i\pi sTl(r))\mathcal{L}_W(2i\pi sTl(r)) \frac{\mathbb{E}\left(e^{2i\pi s\widehat{F}}\right) - 1}{2i\pi s} ds.$$

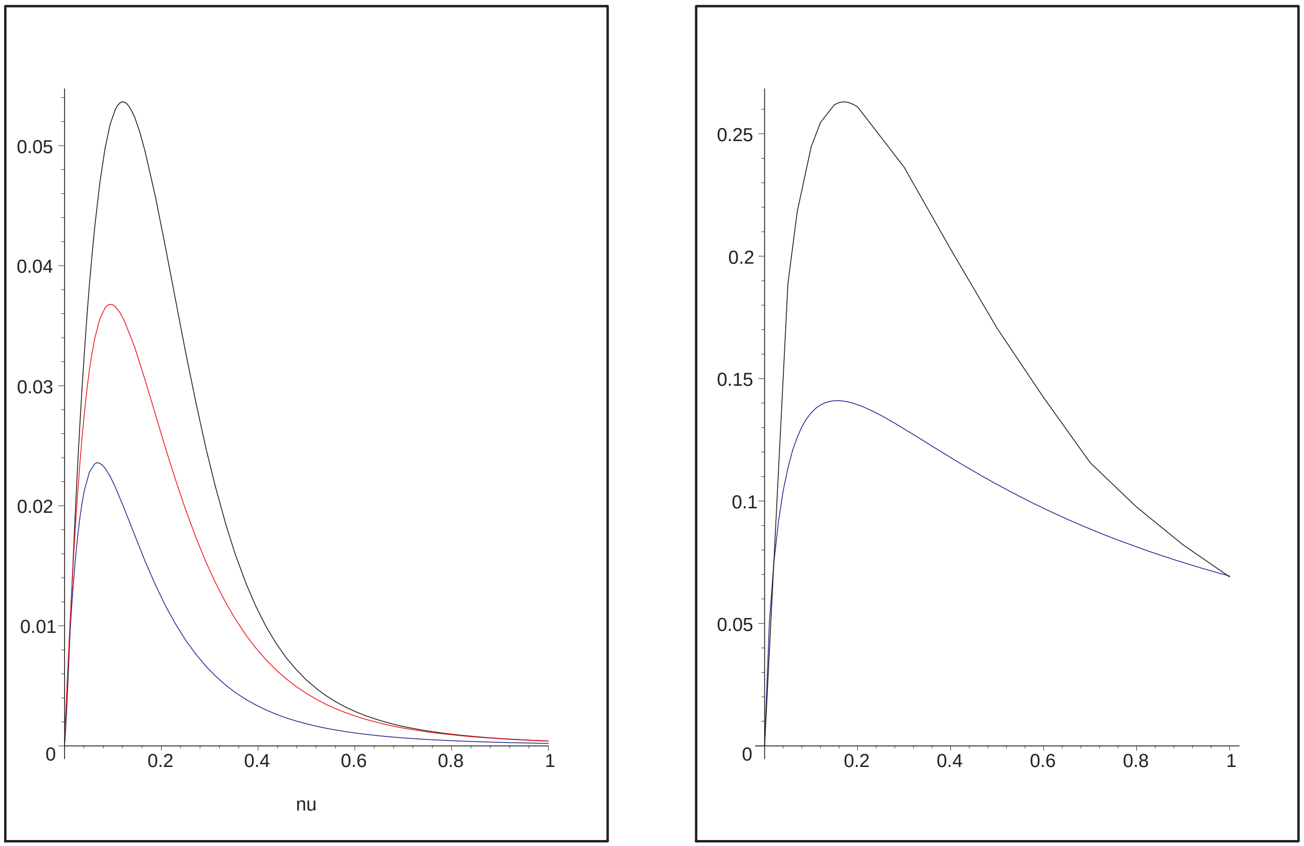


Figure 1: **Left**: Density of success in function of ν in the plain Aloha case with MAP $p = \nu/(\mu + \nu)$ (bottom curve), in Opportunistic Aloha with exponential threshold of parameter ν (middle curve) and in Opportunistic Aloha with deterministic threshold of parameter θ such that $\exp(-\mu\theta) = \nu/(\mu + \nu)$ (top curve). Fading is Rayleigh with parameter $\mu = 1$. **Right**: Transport density in function of p in the plain Aloha case with MAP p (bottom curve) and in Opportunistic Aloha with deterministic threshold of parameter θ such that $\exp(-\mu\theta) = p$ (top curve). Fading is Rayleigh with parameter $\mu = 1$. Here $\lambda = r = 1$ and we use power law propagation with $\beta = 4$.

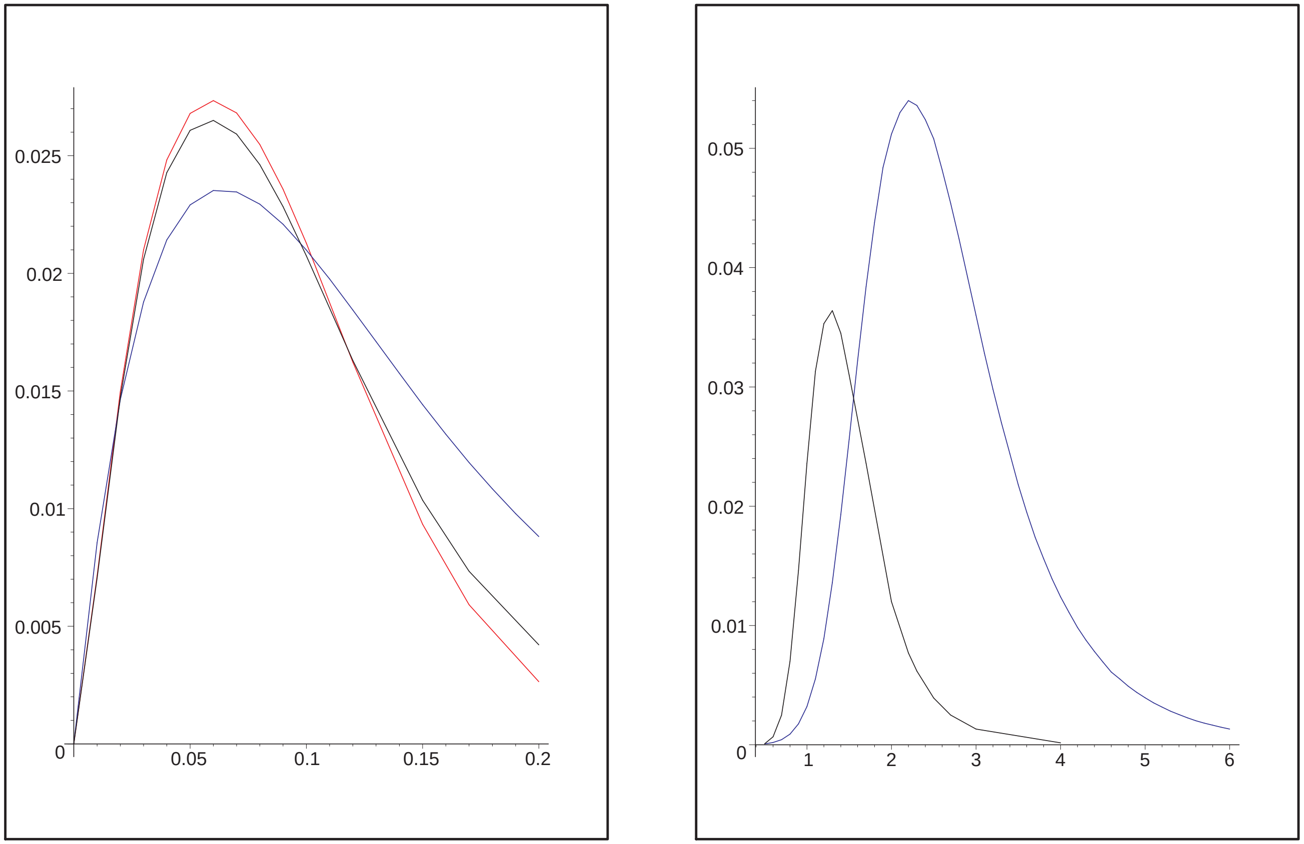


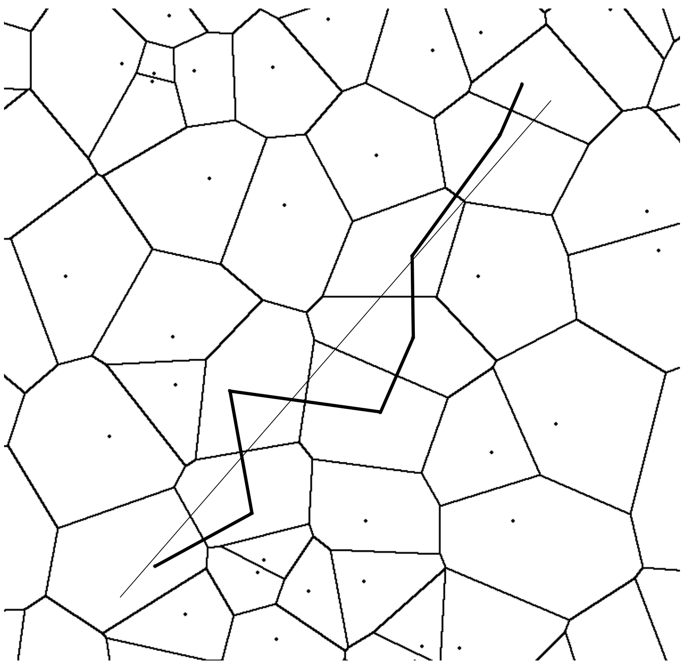
Figure 2: **Left**: Density of success for plain Aloha in function of p in the Rayleigh (in blue) and the Rician (with $q = 1/2$ in black and $q = .9$ in red) cases. **Right**: Density of success for Opportunistic Aloha in function of the threshold θ in the Rayleigh (in blue) and Rician (with $q = 1/2$ in black) cases. In both figures, $\lambda = r = 1$ and $T = 10$dB and we use power law attenuation with $\beta = 4$.

ROUTING

- **Multihop routing on randomly located nodes**
  - **What routing?**
    - **Usual protocols (link state or distance vector) versus geographic protocols**
    - **Point to point versus multicast**
  - **On what wireless links?**
    - **Separation of MAC and Routing: decide of route and ask MAC to implement it**
    - **Interplay of MAC and Routing.**

USUAL ROUTING PROTOCOLS

Distributed algorithms that assume the feasibility of a collection of links and compute an optimal route for any pair of source node and destination node (e.g. OSPF, OLSR).



Ex. links to Delaunay neighbors.

- **A path on the Delaunay graph of a PPP.**
- **Dynamic programming allows one to compute the path with the smallest number of hops between any pair of points (Dijkstra's algorithm).**
- **Requires network state.**

GEOGRAPHIC/GEOMETRIC ROUTING

– **Node positions are used to determine the route from source to destination; in wireless, one should take into account:**

  * **distance to destination;**
  * **distance from transmitter to next relay**

  GPSR (Greedy Perimeter Stateless Routing)
  IGF (Implicit Geographic Forwarding)

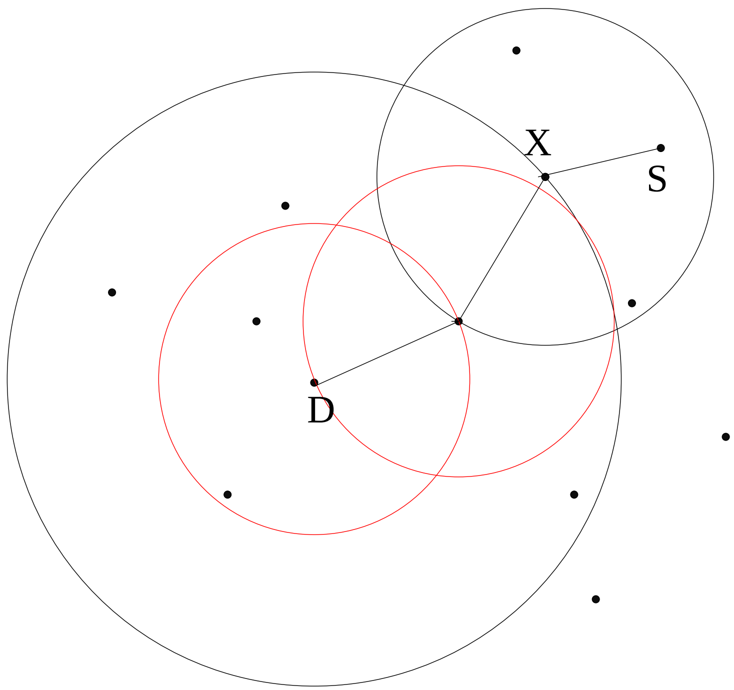
– **Pros:**

  * **Reduction of the node routing state**
  * **Well adapted to highly dynamic settings and to MAC**

– **Cons:**

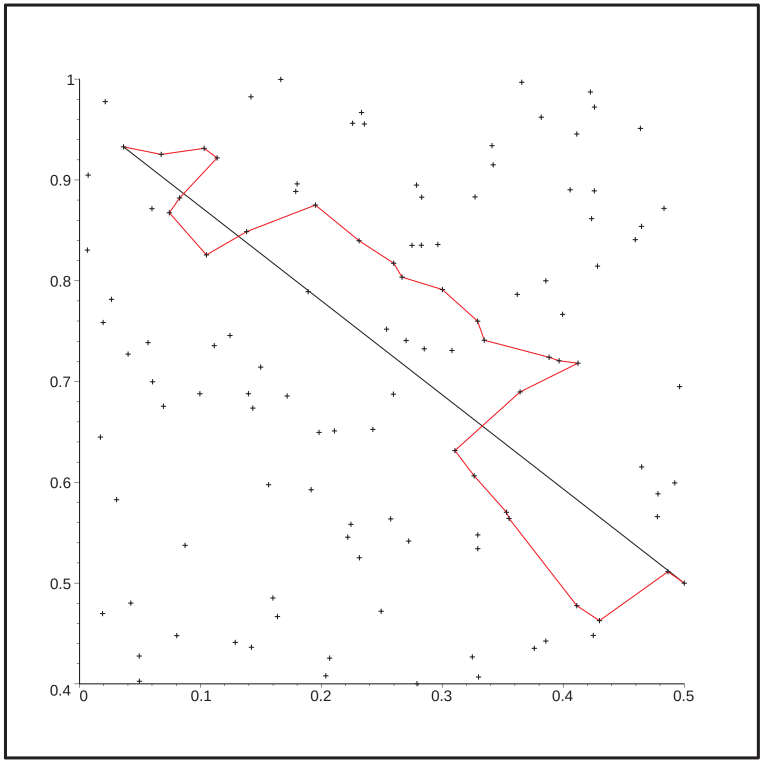
  * **Requires positioning information**
  * **Less optimal compared to shortest path.**

NEAREST NEIGHBOR POINT TO POINT GEOGRAPHIC ROUTING



The next hop on the route from any node X of the route from S to D is the nearest among the nodes which are closer from D than X.

NN GEOGRAPHIC ROUTING OVER A POISSON POINT PROCESS



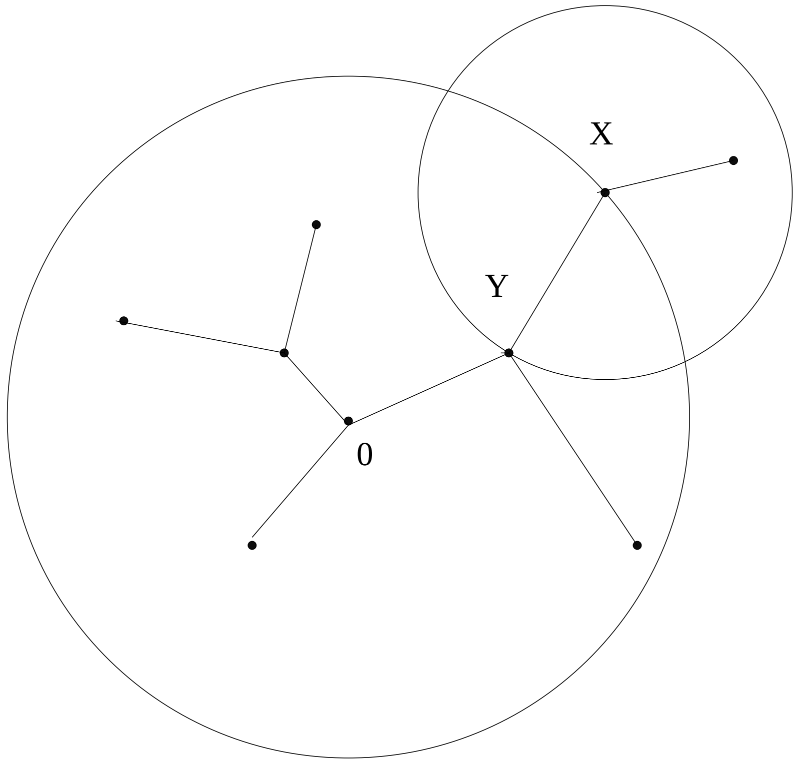
- Distance based geographic routing.
- The path can be built based on local information
- Edges are between neighbors, which is optimal in most wireless networks.

MULTICAST ROUTING

Same Dichotomy:

- **Minimal Spanning Tree: analogue of the shortest path in the point to point case.**
- **Radial Spanning Tree: distance based spanning tree with local construction.**

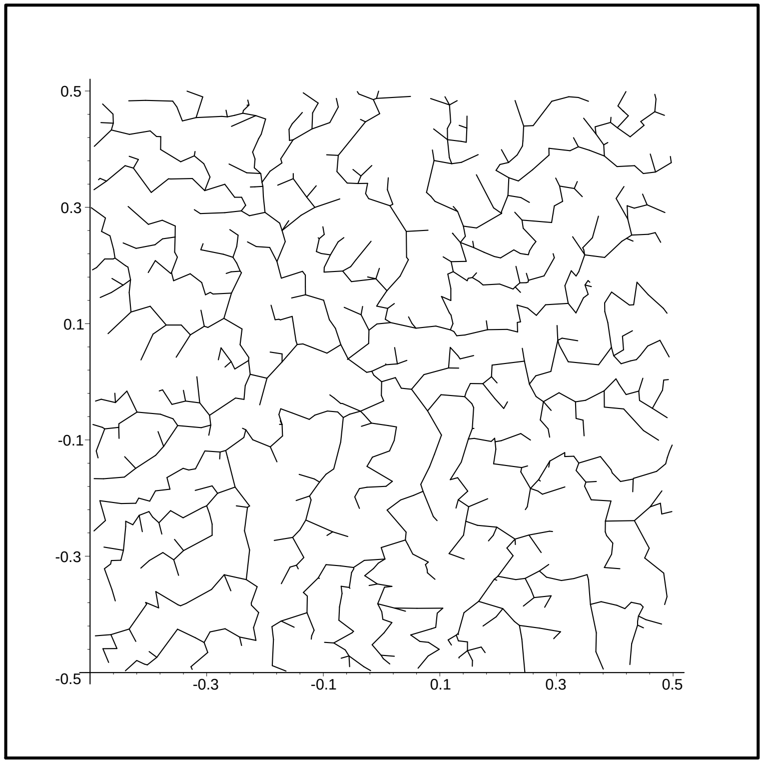
RADIAL SPANNING TREE



The RST with root 0: $\mathcal{T} = (N, E)$ **is a geometric tree on** $N \cup \{0\}$**.**

If $|Y| < |X|$ **and** $X, Y \in N \cup \{0\}$ **then** $(X, Y) \in E \iff N(B(0, |X|) \cap B(X, |X - Y|)) = 0$**.**

RST OVER 1000 RANDOM NODES IN THE UNIT SQUARE



THE GEOMETRY OF NEAREST NEIGHBOR ROUTES

- Spatial averages for length of edges, degree of nodes, progress: analytic expressions using Stochastic Geometry.
- Path averages for e.g. maximal deviation, shape using Harris Markov chain theory.

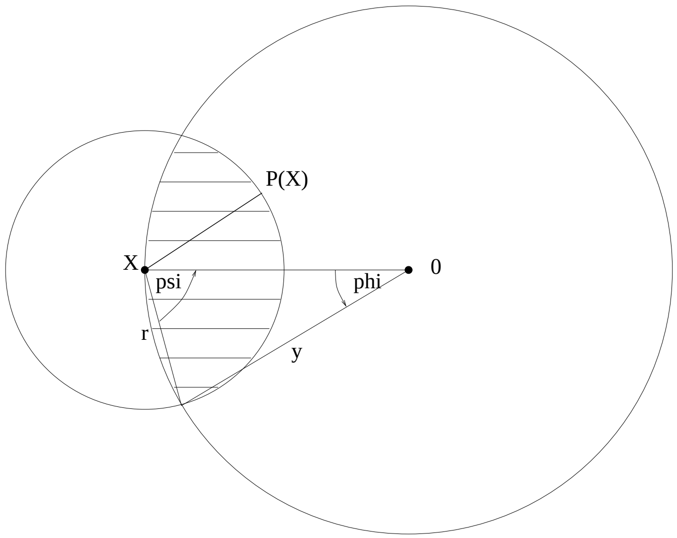
LOCAL FUNCTIONALS: example of the length of an edge

- Let $\mathcal{P}(X)$ be the ancestor of $X \in \mathbb{R}^2$ in the RST (built on $N \cup \{X\}$).
- Let $L(X) = |X - \mathcal{P}(X)|$: length of edge to ancestor.
- $P(L(X) \geq r) = \mathbf{1}(r \leq |X|) P(N(B(X, r) \cap B(0, |X|) = 0)$
  $= \mathbf{1}(r \leq |X|) e^{-M(|X|, r)}$,

where

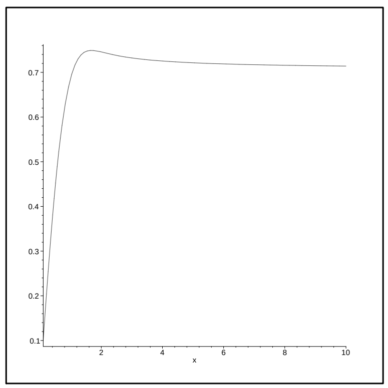
$$M(x,r) = x^2(\phi - \frac{\sin(2\phi)}{2}) + r^2(\frac{\pi}{2} - \frac{\phi}{2} - \frac{\sin(\phi)}{2})$$

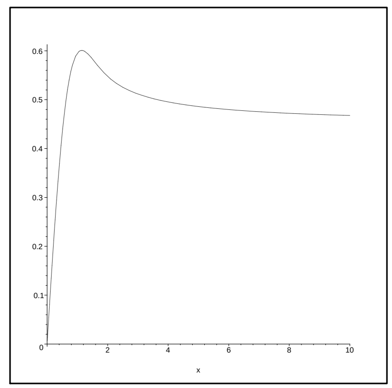
with $\phi = 2 \arcsin \frac{r}{2x}$, is the volume of this lens:



PROGRESS

Same type of computation for the **radial progress:** $P(X) = |X| - |\mathcal{P}(X)|$.





Left: $EL(x)$ **in function of** x. **Right:** $EP(x)$ **in function of** x.

DEGREE OF THE ROOT

Degree of the root $D(0) \leq 5$
Mean degree of the root $ED(0)$

$$D(0) = \sum_{T \in N \setminus 0} \mathbf{1}(N(B(T, |T|) \cap B(0, |T|) = 0).$$

From Campbell's formula:

$$ED(0) \; = \; 2\pi \int_0^\infty e^{-r^2(2\pi/3 - \sin(2\pi/3)} r dr = \frac{\pi}{2\pi/3 - \sqrt{3}/2} \sim 2.56.$$

DEGREE OF A NODE

Degree of node X added to N: $D(X)$

Mean degree of node X: $ED(X)$

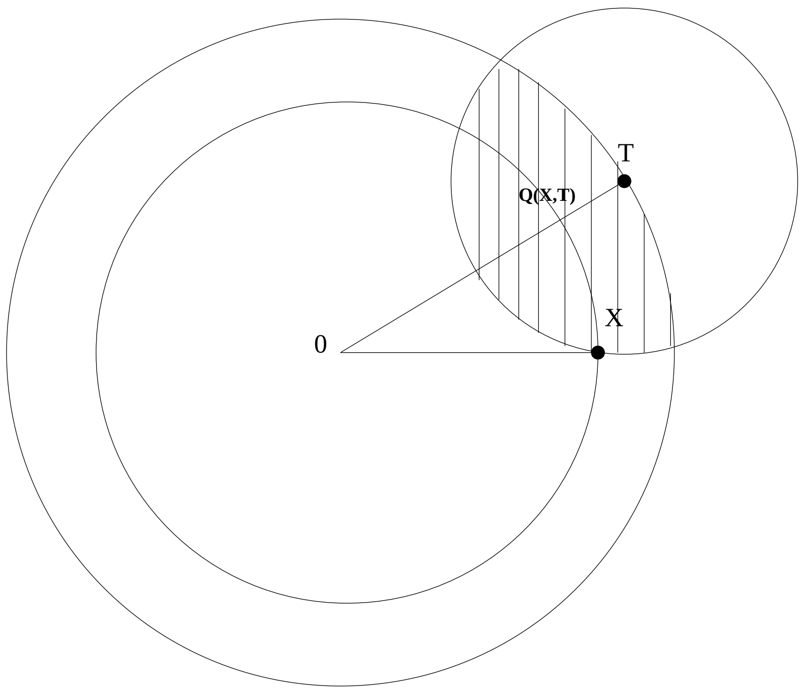
$$D(X) = 1 + \sum_{T \in N} \mathbf{1}(|T| \geq |X|)\mathbf{1}(N(B(T, |X - T|) \cap B(0, |T|) = 0)$$
$$\mathbf{1}(0 \notin B(T, |X - T|)).$$

From Campbell's formula (with $X = (x, 0)$):

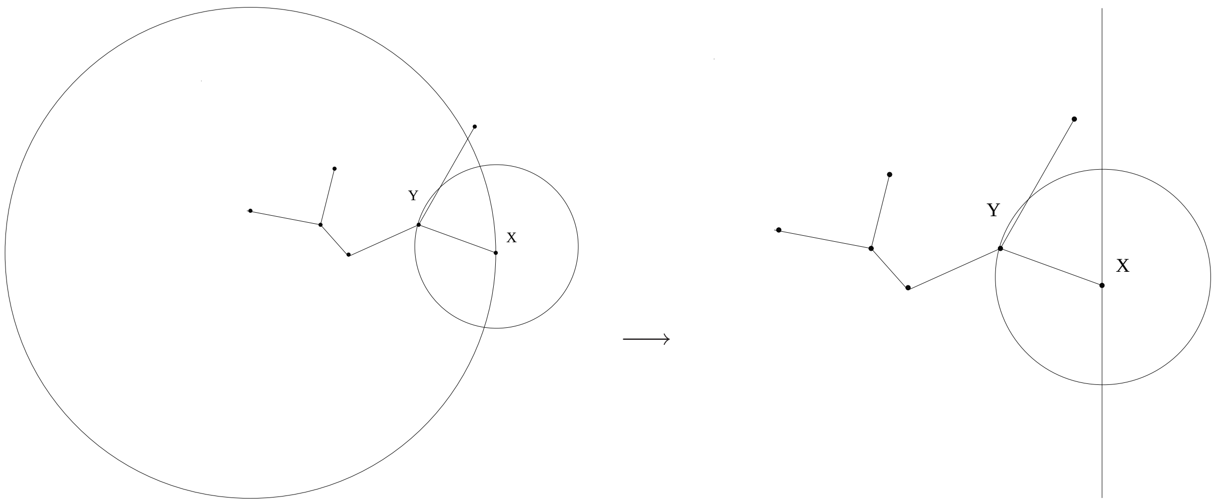
$$\begin{aligned} ED(x) &= 1 + E \sum_{T \in N} \mathbf{1}(N(B(T, |X - T|) \cap B(0, |T|) = 0) \\ &\quad \mathbf{1}(x \leq |T|)\mathbf{1}(|T| > |X - T|) \\ &= 1 + \int_{\rho > x} \int_{-\arccos(\frac{x}{2\rho})}^{\arccos(\frac{x}{2\rho})} e^{-Q(x,\rho,\theta)} \rho d\rho d\theta, \end{aligned}$$

DEGREE OF A NODE (Continued)

$Q(x, \rho, \theta)$ is the surface of the shaded region



LIMIT OF FUNCTIONALS WHEN $|X|$ TENDS TO ∞



$\longrightarrow$ **The ball $B(0, |X|)$ tends locally toward its tangent hyperplane**

Directed Spanning Forest (DSF), $\mathcal{T}_{DSF}$: the ancestor of $x \in N$ is the nearest point of N which has a strictly smaller x-coordinate.

LIMIT OF FUNCTIONALS WHEN $|X|$ TENDS TO ∞

$$Ee^{-sL(x)} \underset{x\to\infty}{\longrightarrow} \int_0^\infty e^{-sr-\frac{\pi r^2}{2}} r dr \quad \textbf{and} \quad EL(x) \underset{x\to\infty}{\longrightarrow} \frac{1}{\sqrt{2}}$$

$$Ee^{-sP(x)} \underset{x\to\infty}{\longrightarrow} \int_0^\infty \int_{-\pi/2}^{\pi/2} e^{-sr\cos\theta-\frac{\pi r^2}{2}} r dr d\theta \quad \textbf{and} \quad EP(x) \underset{x\to\infty}{\longrightarrow} \frac{\sqrt{2}}{\pi}$$

$$ED(x) \underset{x\to\infty}{\longrightarrow} 2$$

SPATIAL AVERAGE : example of edge length

$$L(X) = |X - \mathcal{P}(X)|$$

Total Edge Length : $T_r = \sum_{X \in N} \mathbf{1}(X \in B(0,r)) L(X)$.
Using $EL(x) \to_{x \to \infty} \frac{1}{\sqrt{2}}$ **:**

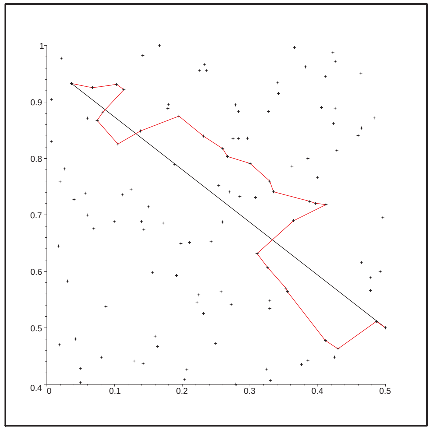
- **Theorem: a.s. and in L^1,**

$$\lim_{r \to +\infty} \frac{T_r}{r^2} = \pi / \sqrt{2}.$$

A similar result holds for $T_r^{(\alpha)} = \sum_{X \in N} \mathbf{1}(X \in B(0,r)) L(X)^\alpha$.

PATH TO THE ROOT IN THE RST

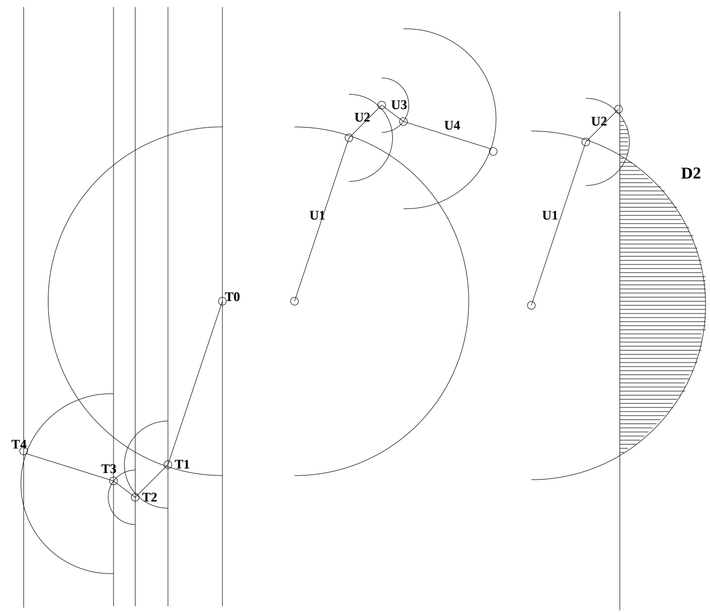
$R_0(X)$:**path from** X **to the root** 0
$H(x)$ **the generation of** x **in the RST and**
$R_0(x) = \{x = T_0, T_1, ..., T_{H(x)} = 0\}$ **the successive ancest. of** x.



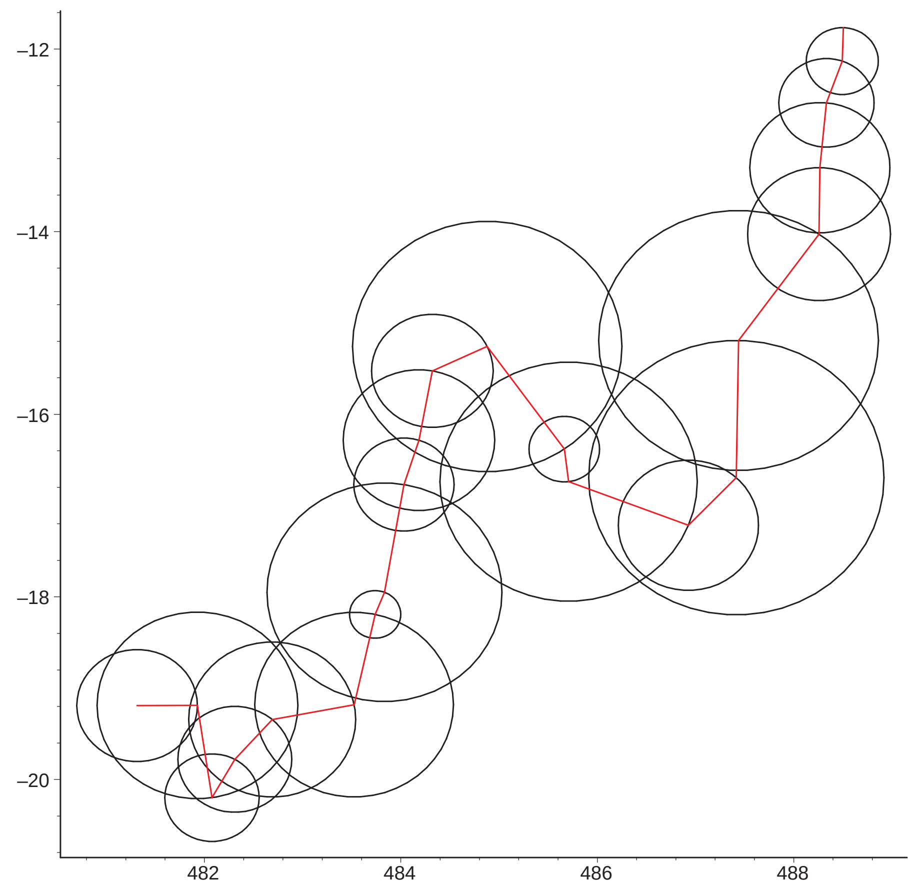
- **Order of Magnitude of the Deviation :** $D_{\max}(x) = d(R_0(x), \overline{0x}) \to ?$
- **Average along the path :** $\frac{H(x)}{x} \to ?, \quad \frac{1}{H(x)} \sum_{k=1}^{H(x)} g(T_{k-1} - T_k) \to ?$

PATH IN THE DSF

$R = \{0 = T_0, T_1, ..., T_n, ...\}$ **the successive ancest. of** 0 **in the DSF.**
The edge process $(U_n = T_n - T_{n+1})$ **is NOT a Markov Chain.**
There exists a sequence of *finite* **stopping times** (τ_k) **such that**



$\Phi_k = (U_{\tau_k}, ..., U_{\tau_{k+1}-1})$ **is a** Ψ**-irreducible, aperiodic Markov Chain which admits a small set and is hence geometrically ergodic.**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482
484
486
488

PATH IN THE DSF

- **Theorem: There exists a measure π such that if $g(x) \leq |x|^\alpha$ for $\alpha > 0$ then a.s. :**

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} g(T_{k+1} - T_k) = \pi(g).$$

In particular there exists constants l_α and p such that:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \langle T_k - T_{k+1}, e_x \rangle = p \qquad \textbf{and} \qquad \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} |T_{k+1} - T_k|^\alpha = l_\alpha.$$

PATH IN THE RST

- **Theorem: If** $g(x) \leq |x|^\alpha$ **for** $\alpha > 0$ **then a.s. :**

$$\lim_{x\to\infty} \frac{1}{H(x)} \sum_{k=0}^{H(x)-1} g(T_{k+1} - T_k) = \pi(g) \quad \textbf{and} \quad \lim_{x\to\infty} \frac{H(x)}{x} = \frac{1}{p}.$$

But

$$p \neq \lim_{x\to\infty} EP(x) = \frac{\sqrt{2}}{\pi} \quad \textbf{and} \quad l_1 \neq \lim_{x\to\infty} EL(x) = 1/\sqrt{2}$$

It follows:

$$\lim_{x\to\infty} \frac{1}{H(x)} \sum_{k=0}^{H(x)-1} |T_k - T_{k+1}| \neq \lim_{r\to\infty} \frac{1}{\pi r^2} \sum_{X \in N \cap B(0,r)} |X - \mathcal{P}(X)|$$

$\longrightarrow$ **The average along a long path is different from the spatial average : averaging along an infinite path creates a bias**

PATH IN THE RST : MAXIMAL FLUCTUATION

The trajectory $R = \{0 = T_0, T_1, ..., T_n, ...\}$ of the successive ancestors of 0 in the DSF converges properly scaled toward a Brownian Motion.
$\longrightarrow$ for the RST, it is not surprising that:

- **Theorem: Almost surely, for all $\epsilon > 0$, the number of points of N such that $D_{\max}(x) = d(R_0(x), \overline{0x}) \geq |x|^{\frac{1}{2}+\epsilon}$ is finite.**

OPPORTUNISTIC ROUTING

– **Routing algorithms**

– **Samples and simulation observations**

– **Joint routing and MAC optimization**

SIR MODEL FOR SPATIAL ALOHA

- $\{X_i\}$ **Poisson Point Process on** $\mathbb{R}^2$ **representing location of nodes**
- $S_{i,j} \in \mathbb{R}^+$: **fading from node** i **to node** j
- $e_i \in \{0,1\}$: **right for** i **to access medium in current slot**
- $W \in \mathbb{R}^+$: **thermal noise**
- $T \in \mathbb{R}^+$: **SIR threshold**
- $l(\cdot)$: **attenuation function**
- $I_{\Phi^e}(j) = \sum_{i \neq 0,j} S_{i,j} e_i l(|X_j - X_i|)$: **interference at** X_j,
- **Node at** X_i **active in slot can be received by that located at** X_j **iff**

$$\frac{S_{i,j} l(||X_i - X_j||)}{W + I_{\phi^e}(j)} \geq T.$$

FADING SCENARIOS

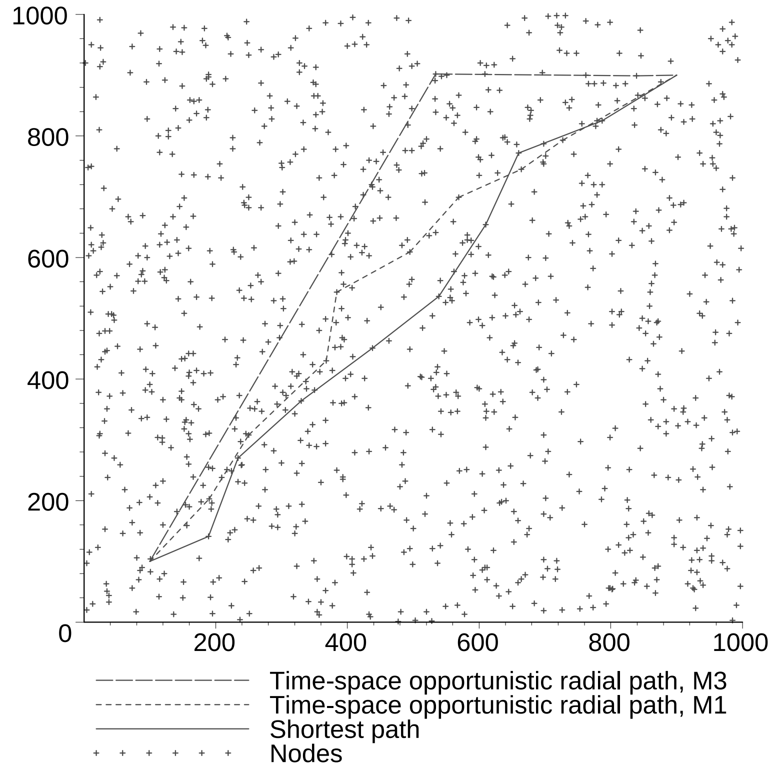
(M1) No fading: all $S_{i,j}$ are equal to 1.

(M2) Position dependent fading: all $S_{i,j}$ are sampled independently for each transmitter-receiver pair and stay constant for all time slots.

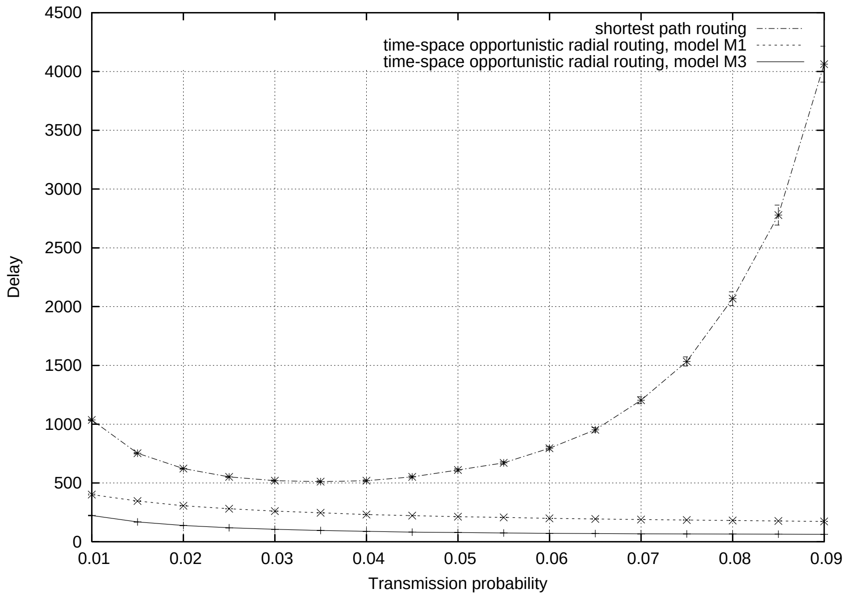
(M3) Position and time dependent fading: all $S_{i,j}$ are sampled independently for each time slot and each transmitter-receiver pair.

For models M2 and M3, we assume a Rayleigh fading

SAMPLES OF OPPORTUNISTIC RADIAL PATHS



SIMULATION BASED OBSERVATIONS

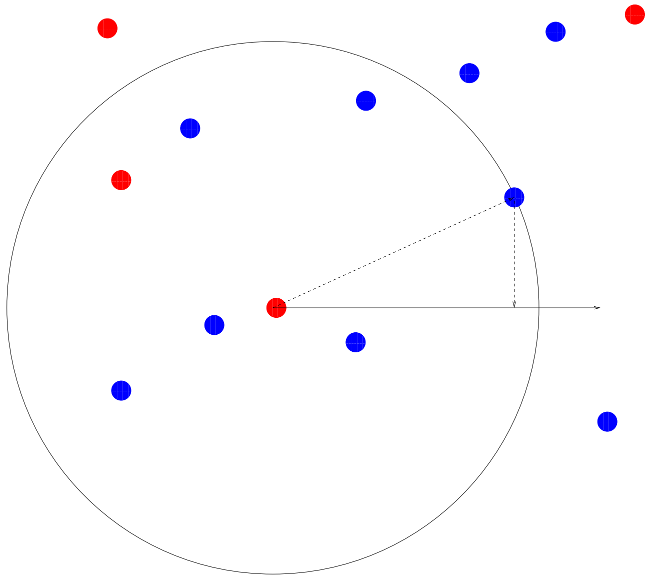


delay versus transmission probability p

The algorithm based on time-space diversity significantly outperforms the conventional shortest path routing strategy: the average delay of a packet is at least twice as small for this strategy than for Dijkstra's algorithm.

OPPORTUNISTIC DIRECTIONAL ROUTING: PROGRESS-1

Φ^1: **intensity** λp **transmitters**, Φ^0: **intensity** $\lambda(1-p)$ **receivers**



Progress-1 best progress to destination achieved in one hop:

$$D = \max_{X_j \in \Phi^0} \left(1_{\frac{S_{0,j} l(|X_j|)}{W + I_{\Phi^1}(X_j)} \geq T} |X_j| \Big(\cos(\arg(X_j)) \Big)^+ \right),$$

Maximization of the density of progress:

$$\lambda p d(\lambda, p)$$

with $E[D] = d(\lambda, p)$**. Equivalent to minimization of end to end delay.**

SPATIAL AVERAGES OF PROGRESS

Individual Progress-2 surrogate of progress-1 used in following calculations

$$\tilde{D} = \max_{X_j \in \Phi^0} \left(\left(p_{|X_j|}(\lambda p) |X_j| \left(\cos(\arg(X_j)) \right)^+ \right) \right),$$

$E[\widetilde{D}] = \widetilde{d}(\lambda, p)$

For all λ, p, $d(\lambda, p) \geq \tilde{d}(\lambda, p)$.

OPTIMAL MEDIUM ACCESS PROBABILITY

An extremal shot-noise $\max_{X_i\in\Phi^0} g(X_i)$**, with** Φ^0 **Poisson has a distribution which can be obtained from the Laplace transform of the (additive) shot noise via:**

$$P(\max_{X_i\in\Phi^0} g(X_i) \le z) = \mathbb{E}\left[\exp\left[\sum_{X_i\in\Phi^0} \ln(\mathbf{1}(g(X_i) \le z))\right]\right]$$

Since $\tilde{D}$ **is an extremal shot-noise with response function**

$$g(x) = p_{|x|}|x|(\cos(\arg(x)))^+$$

w.r.t. a Poisson PP of intensity $\lambda(1-p)$**,**

$$P(\tilde{D} \le z) = \exp\left[-\lambda(1-p)\int_{\mathbb{R}^2} \mathbf{1}(g(x) > z)\,dx\right].$$

DISTRIBUTION OF PROGRESS IN THE M/M CASE

Theorem In the M/M case,

$$F_{\tilde{D}}(z) = P(\tilde{D} \leq z) = e^{-\lambda(1-p)(r_{\max}(\lambda p))^2 G(z/\rho(\lambda p))} .$$

with $p_r = e^{-\lambda p r^2 T^{2/\beta} C}$,

$$r_{\max}(\lambda) \; = \; \mathbf{maxarg}_r\{r p_r(\lambda)\} = \frac{1}{T^{1/\beta}\sqrt{2\lambda C}},$$

$$\rho(\lambda) \; = \; \max_r\{r p_r(\lambda)\} = \frac{r_{\max}(\lambda)}{\sqrt{e}}$$

MEAN INDIVIDUAL PROGRESS (2)

THEOREM The maximal spatial density of progress is attained for MAP p^*, which does not depend on λ, satisfying

$$\int_0^1 \left(1 + \frac{G(z)}{p^* T^{2/\beta} C}\right) \exp\left[\left(1 - \frac{1}{p^*}\right) \frac{G(z)}{2T^{2/\beta} C}\right] dz = 1 \,.$$

with

$$G(z) \;=\; 2 \int_{\{t: e^t/\sqrt{2et} \le 1/z\}} \arccos\left(\frac{ze^t}{\sqrt{2et}}\right) dt \,. \quad z \in [0,1]$$

Same type of result for Progress 1 by scaling arguments.

MEAN INDIVIDUAL PROGRESS (2)

THEOREM **Under the assumptions of Rayleigh fading, if** $l(r) = (Ar)^{-\beta}$ **and** $W \equiv 0$**, then**

$$\widetilde{d}(\lambda, p) = \mathbb{E}[\tilde{D}(\lambda, p)] = \frac{1}{\sqrt{\lambda p} T^{1/\beta} \sqrt{2eC}} H(p)$$

with

$$H(p) = \int_0^1 1 - \exp\left[\left(1 - \frac{1}{p}\right) \frac{G(z)}{2T^{2/\beta} C}\right] dz$$

- **For all values of** p **(an in particular for** p^***), Opportunistic Aloha achieves the upper bound capacity of Gupta and Kumar.**

CONCLUSIONS

– **Interference can be represented by shot noise fields**

– **Fourier analysis allows one to analyze coverage and throughput offered by MAC protocols**

– **SG might in the long run play for wireless networks a role similar to that of Queuing Theory in wired networks: compute macroscopic space/time averages.**

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